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| February 26, 2015 | Matt Landreman |

# Equations implemented in the minimal solver for stellarator monoenergetic transport coefficients

This code solves the following normalized kinetic equation:

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This is a 3D linear inhomogeneous equation for . The independent variables  and  are periodic spatial coordinates (with period ), and the independent variable  is the cosine of the pitch angle in velocity space. The domain of  is . In ,  is a constant scaling the diffusive term. The constants , , and  are related to the geometry of the magnetic field. The magnetic field strength  is a specified function. The model used in the code is



where  is called  in the code and  is called . Since the system has a discrete symmetry in , the code only simulates the domain  rather than the full .

For a discretization of the  coordinate that is both sparse and spectrally accurate, the unknown  is expanded in Legendre polynomials :

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We discretize by applying the operation

.

Thus, becomes



where the main linear operator has become



and the right-hand side is

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The PDE has a single null solution, constant. We eliminate this null solution by imposing one extra equation in an additional row of the matrix. For physical reasons, we choose this condition to be

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In the Legendre modal expansion, becomes

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To keep the linear system square, we also add one extra unknown, a number , which is added to the left-hand side of . ( is independent of all 3 independent variables.) Equivalently, to the left-hand side of we add . We can think of  as a source in the kinetic equation , and it plays a role like a Lagrange multiplier. By applying the operation



to and integrating by parts in several places, it can be seen that all terms vanish or cancel except , forcing . Thus, we are not really changing our original equation by introducing , since it will always end up vanishing.

Combining and , our final linear system has the  block structure

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